Decoherence and the (non)emergence of classicality

Steven Weinstein[*]

Perimeter Institute for Theoretical Physics,

31 Caroline St N, Waterloo ON N2L 2Y5 Canada

(Received October 24, 2008)

We consider the claim that decoherence explains the emergence of classicality in quantum systems, and conclude that it does not. We show that, given a randomly chosen universe composed of a variety of subsystems, some of which are macroscopic and subject to decoherence-inducing interactions, and some of which are microscopic, the macroscopic subsystems will not display any distinctively classical behavior. Therefore, a universe in which macroscopic and microscopic do display distinct behavior must be in a very special, highly nongeneric quantum state.

I.   INTRODUCTION

Over the last four decades [1], the study of decoherence has begun to shed light on the effects of the interaction of open quantum systems with their environments. It has been shown that, for some interesting model systems, certain pure states, sometimes called *pointer states* [2] [3] survive interaction with the environment without losing their coherence or purity, while superpositions of these states lose coherence in such a way that the result is an incoherent, improper *mixture* of such states which is subsequently approximately stable.

The fact that macroscopic subsystems interacting with an appropriate environment can be seen to exhibit decoherence in a preferred basis, along with the fact that the basis in question often corresponds to a paradigmatically classical observable such as position, has led to claims that "the classical structure of phase space emerges from the quantum Hilbert space in the appropriate limit" [4]; that "the appearance of classicality is therefore grounded in the structure of the physical laws governing the system-system environment interactions"[5]; and that "there are strong signs that the transition [from quantum to classical] can be understood as something that emerges quite naturally and inevitably from quantum theory"[6]. Other, similar claims lie ready to hand [7][8]. In this paper, we show that the properties of generic microscopic subsystems of a quantum-mechanical universe are kinematically and dynamically indistinguishable from the properties of

---
[*]Electronic address: sweinstein@perimeterinstitute.ca; sw@uwaterloo.ca

generic macroscopic subsystems, and thereby show that decoherence does not explain the emergence of (quasi)classicality.

II.   DECOHERENCE, EINSELECTION, AND QUASICLASSICALITY

Consider a subsystem S with (pure) state ψ^S interacting with an environment E with state ψ^E. If the subsystem is sufficiently macroscopic, and if the Hamiltonian governing the combined evolution of subsystem and environment is appropriate, then the environment as a whole acts as a kind of measuring device, in that the effective state of the environment (given by its reduced density matrix) will reliably become correlated with certain subsystem observables. Which properties of the system are "measured" by the environment – which observables (if any) become nontrivially correlated – will depend on the Hamiltonian, including the self-Hamiltonians of system and of environment [9]. Eigenstates of the subsystem observables in question, the pointer states, will be stable or approximately stable under such measurement-like interactions, while arbitrary *superpositions* of pointer states will evolve into improper *mixtures* of those states as a result of the environment's correlation with the pointer observable. The tendency for the reduced density matrix of the subsystem to be driven into a small subset of the available states by the environment is called *einselection*, short for *environment induced superselection* [3][4].

Decoherence, then, refers to the process by which pure states lose their coherence, and more particularly to the way in which sufficiently macroscopic subsystems lose their coherence in a way characterized by einselection. Subsystems which undergo einselection said to be quasiclassical (sometimes simply "classical") in virtue of their stability and predictability; they not only lose coherence – so do many nonclassical, microscopic systems – but they do so in a predictable way, and evolve stably thereafter. (The qualification "quasi" is in place because an improper mixture has no direct classical analog, and because einselection is never exact and is subject to Poincare recurrences.)

A.   Example: Central spin model

Consider for example the so-called central spin model [3], in which one contemplates a system consisting of $N+1$ two-level systems, N of which are coupled to a central spin S via the Hamiltonian

$$\hat{H} = \frac{1}{2}\hat{\sigma}_z \otimes \left(\sum_{i=1}^{N} g_i \hat{\sigma}_z^{(i)} \bigotimes_{i' \neq i} \hat{I}_{i'} \right)$$

where $\hat{I}_i$ is the identity operator for the i'th system. (Here there is no macroscopic/microscopic distinction; rather, the distinctive dynamical role of the central spin singles it out as special.) An initial pure state of the form

$$\psi = \alpha \left|+z\right\rangle \left|E_0\right\rangle + \beta \left|-z\right\rangle \left|E_0\right\rangle$$

will, via the unitary evolution $U(t) = e^{-i\hat{H}t}$ generated by this Hamiltonian, evolve toward an entangled state $\psi(t) = \alpha \left|+z\right\rangle \left|E_+(t)\right\rangle + \beta \left|-z\right\rangle \left|E_-(t)\right\rangle$. After a sufficient amount of time t_d has passed, $\langle E_+|E_-\rangle \approx 0$, and the reduced density matrix of the central spin will be well-approximated by

$$\rho^S = \alpha^2 \left|+z\right\rangle \left\langle+z\right| + \beta^2 \left|-z\right\rangle \left\langle-z\right|.$$

One can represent this evolution on the Bloch sphere as the evolution of initially pure states of the central qubit (the surface of the sphere), evolving, modulo extremely unlikely Poincare-type fluctuations, toward a narrow ellipse along the z axis:
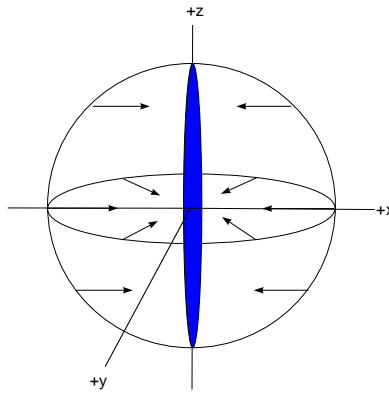
Evolution of the central spin

Though it is no surprise that a subsystem should lose coherence upon interaction with an environment, and thus move away from the surface of the Bloch sphere, we have here in addition the phenomenon of einselection, in which the loss of coherence is in a preferred direction. In particular, the loss of purity is proportional to the angle with the z axis, with the pointer states $\left|+z\right\rangle$ and $\left|-z\right\rangle$ suffering no loss whatsoever.

III.   PROPERTIES OF GENERIC SUBSYSTEMS

Consider a large system described by an arbitrary pure state $\rho = \left|\psi\right\rangle \left\langle\psi\right|$ defined over a Hilbert space $\mathcal{H}$ which can be decomposed into a tensor-product of Hilbert spaces $\mathcal{H}_i$, one of which corre-

sponds to the subsystem S of interest and the rest of which collectively correspond to the environment E. Thus

$$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_2 \otimes \mathcal{H}_3 \otimes ... \otimes \mathcal{H}_k. \tag{1}$$

The state $|\psi\rangle \in \mathcal{H}$ with density matrix $\rho = |\psi\rangle\langle\psi|$ is then just a vector in this space, and the state of S, $\rho_S = Tr_E(\rho)$, is just the partial trace of ρ over the environment E. The claim we are assessing is that, given the appropriate Hamiltonian, subsystems S which are appropriately macroscopic (or possessed of other properties which make them suitable candidates for quasiclassicality) undergo einselection and subsequently evolve in such a way as to exhibit stability in a preferred basis of pointer states, thus dynamically distinguishing them from the states of microscopic systems. We now show that this is not the case.

A. Kinematics

For simplicity, we restrict attention to finite-dimensional Hilbert spaces, as in eqn. (1). The physical states are the unit vectors in this space, and an unbiased probability measure on these states is naturally defined via the Haar measure over $SU(n)$, where $n = \dim(\mathcal{H})$. This measure has the desirable property that no particular basis is privileged. The ensemble of equiprobable states is given by the density matrix

$$\Omega = \sum_{i=1}^{n} \frac{1}{n} |\psi_i\rangle\langle\psi_i| \tag{2}$$

where the $|\psi_i\rangle$ constitute an orthonormal basis for the Hilbert space. The rotational invariance of the measure means that this density matrix looks the same in any basis; it is simply a multiple of the identity, with $Tr(\Omega) = 1$.

Let us consider what one can say about the typical properties of our arbitrary pure state $\rho = |\psi\rangle\langle\psi|$. In particular, we would like to ask about the properties of the reduced density matrices $\rho_S = Tr_E(\rho)$ of subsystems S of dimension $m << n$. Popescu *et al* [10] show that, for the vast majority of states, $\rho_S \approx Tr_E(\Omega) \equiv \Omega_S$. More specifically, they use Levy's lemma to exhibit a bound

$$0 < \langle D(\rho_S, \Omega_S) \rangle \leq \frac{m}{2}\sqrt{\frac{1}{n}} \tag{3}$$

on the average distinguishability of ρ_S and Ω_S, where $D(\rho,\Omega) := \frac{1}{2}Tr\sqrt{(\rho-\Omega)^\dagger(\rho-\Omega)}$. Since $D(\rho,\Omega)$ is by definition always positive, the smallness of the *average* distinguishability $\langle D(\rho_S, \Omega_S) \rangle$

implies that, for the vast majority of subsystem density matrices ρ_S, $D(\rho_S, \Omega_S)$ is small as well. As long as the dimension m of a subsystem is much smaller than the total Hilbert space dimension n, then for almost all states ρ, the properties of S will be essentially indistinguishable from the ensemble average for S, which is Ω_S. Since Ω is a multiple of the identity, so too is $\Omega_S = Tr_E(\Omega)$, which means that an arbitrarily chosen small subsystem *behaves as if it were described by a maximally mixed state*, which is in turn to say it behaves randomly with respect to any choice of observable. *A fortiori,* generic subsystems exhibit no preferred basis. E.g., the density of states for the central spin has the form $\varrho(r) \approx (1 - r^2)^{n-2}$, where n is the dimension of the environment and where $0 \leq r \leq 1$ is the radius of the Bloch sphere [11].

B. Dynamics

Of course the decoherence program does not insist that generic states ρ_S have any special properties. Rather, the suggestion is that systems which are prone to decoherence and einselection *evolve* in such a way that they *become* quasiclassical. We now show that the vast majority of subsystems, microscopic or macroscopic, begin in and remain in a nearly maximally-mixed state. If such states are deemed quasiclassical (in virtue of their stability), then almost all subsystems are quasiclassical. If they are not deemed quasiclassical (because they have maximal von Neumann entropy and carry no information), then almost no states, macroscopic or microscopic, are quasiclassical. In either case, there is no difference between the behavior of a typical macroscopic and a typical microscopic subsystem.

Suppose that after some time t_d, the density matrices $\rho'_S = Tr_E(e^{iHt_d}\rho e^{-iHt_d})$ for the subsystems S have become significantly distinct, on average, from the maximally mixed state Ω_S. This implies that $\langle D(\rho'_S, \Omega_S) \rangle$ can exceed the initial bound $\frac{m}{2}\sqrt{\frac{1}{n}}$. But it is readily seen that this *cannot* be the case, no matter what form the Hamiltonian takes. For just as the classical Liouville theorem informs us that a uniform distribution on the phase space will remain uniform over time, the quantum analogue of the theorem tells us that our initially uniform distribution Ω will remain constant under the unitary evolution $U = e^{iHt_d}$ [12]. In particular,

$$\Omega \overset{U(H,t_d)}{\rightarrow} \Omega' = \Omega \ . \tag{4}$$

Since $\Omega_S = Tr_E(\Omega) = Tr_E(\Omega')$, we have

$$\Omega_S \rightarrow \Omega'_S = \Omega_S. \tag{5}$$

(The evolution of the density matrices for the subsystems is not unitary, of course, but it is induced by the unitary evolution for the system as a whole.) We know that, as in equation (3),

$$0 < \left\langle D(\rho'_S, \Omega'_S) \right\rangle \leq \frac{m}{2}\sqrt{\frac{1}{n}}, \tag{6}$$

and using equation (5) we can write

$$0 < \left\langle D(\rho'_S, \Omega_S) \right\rangle \leq \frac{m}{2}\sqrt{\frac{1}{n}}. \tag{7}$$

Thus the bound on the average distinguishability of ρ_S from maximally mixed Ω_S is *constant* in time, and so there is no evolution toward a preferred basis.

If the emergence of quasiclassicality means the emergence of a preferred basis for generic, suitably macroscopic subsystems, then decoherence does not explain the emergence of quasiclassicality. If on the other hand it simply means stability of generic subsystems, then we certainly have that, for both microscopic and macroscopic subsystems, since the maximally mixed states which dominate the ensemble are stable over time. However, this sort of stability has nothing to do with the choice of Hamiltonian, and it is not specific to macroscopic subsystems. In fact, from an information-theoretic standpoint, it would seem to be a rather trivial kind of stability. Given that it means that the subsystem is overwhelmingly likely to be in a state of maximal (von Neumann) entropy, the stability of the state of the subsystem simply reflects the fact that it is a subsystem about which one knows nothing and about which one continues to know nothing over time.

What decoherence *does* tell us is that *if* we have a subsystem with suitable properties which is *not* in a maximally mixed state, which is interacting in an appropriate way with its environment, it will evolve into a stable, quasiclassical state if it is not already in one. Thus for the central spin in the example above, we can say with a high degree of certainty that the rare spin which begins in a state away from the origin of the Bloch sphere will evolve to, and remain in, a state which is well-approximated by an improper mixture of $|+z\rangle$ and $|-z\rangle$ states. But note, too, that the time-reversibility of the dynamics tells us that with overwhelming probability, this rare spin state must also have come *from* such a mixture in the moments immediately prior. Thus decoherence explains how very special, highly non-generic states manage to maintain their quasiclassicality. It does not explain how quasiclassicality "emerges" for generic subsystems.

IV.   DISCUSSION

The main feature that emerges from our discussion is that the states of generic subsystems are stable, be they macroscopic or microscopic, and thus that the distinctively classical behavior of

macroscopic subsystems is still in need of explanation. Certainly, atypical macroscopic subsystems may exhibit a dynamical behavior which is distinct from the dynamical behavior of their microscopic counterparts. But such subsystems are far from generic.

The fact that distinctive behavior only emerges for subsystems starting in highly atypical states suggests a connection with the second law of thermodynamics. Indeed, it has been suggested, in the context of the decoherent *histories* framework [13] that the emergence of classicality is indeed a function of very special initial conditions, and that the same constraints on initial conditions which yield an arrow of time (i.e. a monotonic increase in thermodynamic entropy) are those that lead to the emergence of classical behavior [14][15]. This is clearly a matter worthy of further investigation, and furthermore a matter which is likely to shed light on the relation between the open-systems decoherence models studied here and the closed system decoherent-histories models studied elsewhere.

---

[1] H. Zeh, Foundations of Physics **1**, 69 (1970).

[2] W. Zurek, Phys. Rev. **D24**, 1516 (1981).

[3] W. Zurek, Phys. Rev. D **26**, 1862 (1982).

[4] W. Zurek, Rev. Mod. Phys. **75**, 715 (2003).

[5] M. Schlosshauer, *Decoherence and the Quantum-to-Classical Transition* (Springer, Berlin, 2007).

[6] P. Ball, Nature **453**, 22 (2008).

[7] W. Zurek, in *Conceptual Problems of Quantum Gravity*, edited by A. Ashtekar and J. Stachel (Birkhäuser, Boston, 1991), pp. 43–66.

[8] E. Joos, H. Zeh, C. Kiefer, D. Giulini, J. Kupsch, and I.-O. Stamatescu, *Decoherence and the Appearance of a Classical World in Quantum Theory* (Springer-Verlag, Berlin, 2003), 2nd ed.

[9] C. Dawson, A. Hines, R. McKenzie, and G. Milburn, Phys. Rev. A **71**, 052321 (2005).

[10] S. Popescu, A. J. Short, and A. Winter, Nature Physics **2**, 754 (2006).

[11] K. Zyczkowski and H.-J. Sommers, J. Phys. pp. 7111–7125 (2001).

[12] R. Tolman, *The Principles of Statistical Mechanics* (Dover, New York, 1980).

[13] M. Gell-Mann and J. Hartle, in *Complexity, Entropy and the Physics of Information, SFI Studies in the Science of Complexity, Vol. VIII*, edited by W. Zurek (Addison-Wesley, Reading, 1990), pp. 425–458.

[14] J. Hartle, in *Proceedings of the Lanczos International Centenary Conference*, edited by J. Brown, M. Chu, D. Ellison, and R. Plemmons (SIAM, Philadelphia, 1994).

[15] J. Hartle (2008), arXiv:0806.3776v1.